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A Family of Ratio-Type Estimators of Finite Population Variance

Jamiu Olasunkanmi Muili[1*], Eric Ndukaku Agwamba[1], Yahqub Ayinde Erinola[1], Mojeed Abiodun Yunusa[2], Dr. Ahmed Audu[2]and Muslihu Adeyemi Hamzat[2]

[1]*Department of Mathematics, Kebbi State University of Science and Technology Aliero, Nigeria*
[2]*Department of Mathematics, Usmanu Danfodiyo University, Sokoto, Nigeria.*

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ABSTRACT
This paper deals with the estimation of finite population variance. A class of ratio-type estimators is proposed for estimation of finite population variance of study variable. The purpose of this study is to develop a new class of ratio-type estimators to improve the precision of estimation of finite population variance in sample random sampling without replacement using information of auxiliary variable. The expressions of the bias and mean square error (MSE) of the proposed estimators were derived up to first degree of approximation by Taylor series method. The efficiency conditions under which the proposed ratio-type estimators are better than sample variance, ratio estimator, and other estimators considered in this study have been established. The empirical results shown that the proposed estimators are more efficient than the sample variance, Isaki ratio estimator and other existing estimators.

KEYWORDS: Sampling survey, Ratio estimator, Mean Square Error, Percentiles.

I. INTRODUCTION

Sampling survey is a technique that deals with the estimation of population parameters (mean, total or variance population) under consideration. Percentiles divide a set of ordered data into hundredths. Percentiles play an important part in descriptive statistics and their use is well recommended. Appropriate use of auxiliary information result to reduction in variance or mean square error of the estimator.In a situation where the information about an auxiliary variable X is known and the association between the study variable and the auxiliary variable is positive, the ratio estimation method is useful. Then if the relationship is negative, the product estimation method can be used efficiently.So, to determine the most efficient estimator in a set of estimators, estimator with least value of variance or mean square error is considered as the best.Estimation of population variance has been extensively discussed by many researchers so as to improve and increase the precision of an estimate under consideration. The problem of estimating population variance of the study variable when the population variance of an auxiliary variable(s) is/are known has been discussed among the statisticians in the field of sample survey. The early work on the estimation of population variance was initiated from the work of [1], [2] and [3]. [3] developed a ratio estimator of population variance of study variable for estimation of finite population variance.[4] improved the work of [3] by imposing coefficient of kurtosis to the work of [3]. Since then many researchers have been improving and modifying estimators of population variance using auxiliary information in one way or the other researchers like [5],[6], [7], [8], [9], [10], [11], [12], [13], [14]. [15], etc.

The purpose of this study is to develop a new class ratio-type estimators to improve the precision of estimation of population variance in sample random sampling without replacement using available information of auxiliary variable.

Consider a finite population $U=\{U_1,U_2,...,U_N\}$ having N units where each $U_i=(X_i, Y_i),\ i=1,2,...,N$ has a pair of values. X is the auxiliary variable which Y is the study variable and is correlated with X, where $y=\{y_1,y_2,...,y_n\}$ and $x=\{x_1,x_2,...,x_n\}$ are the n sample values. $\bar{y}$ and $\bar{x}$ are the sample means of the study and auxiliary variables respectively. Let S_y^2 and S_x^2 be the population mean squares of Y and X respectively and s_y^2 and s_x^2 be respective sample mean squares based on the random sample of size n drawn without replacement. N: Population size, n: Sample size, Y: Study variable, X: auxiliary variable, $\bar{y}, \bar{x}$: Sample means of study and auxiliary variables, $\bar{Y}, \bar{X}$: Population means of study and auxiliary variables, f: Sampling fraction, ρ: Coefficient of correlation, C_y, C_x: Coefficient of variations of study and auxiliary variables, Q_3: The upper quartile, QD: Population Quartile Deviation, β_1: Coefficient of skewness of auxiliary variable, β_2: Coefficient of kurtosis of auxiliary variable, TM: Tri-Mean, M_d: Median of the auxiliary, MR: Population mid-range, M_x: Maximum value of auxiliary variable, HL: Hodges-Lehman estimator, G: Gini's Mean Difference, D: Downton's Method and P_i: Percentiles.

$$\bar{X} = \frac{1}{N}\sum_{i=1}^{N} X_i, \quad \bar{Y} = \frac{1}{N}\sum_{i=1}^{N} Y_i, \quad \bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i, \quad \bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i, \quad \gamma = \frac{1-f}{n}, \; f = \frac{n}{N},$$

$$TM = \frac{(Q_1 + 2Q_2 + Q_3)}{4}, s_y^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2, \quad s_x^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2, \; S_y^2 = \frac{1}{N-1}\sum_{i=1}^{N}(Y_i - \bar{Y})^2,$$

$$S_x^2 = \frac{1}{N-1}\sum_{i=1}^{N}(X_i - \bar{X})^2, \; MR = \frac{X_{(1)} + X_{(N)}}{2}, \; HL = Median\left(\frac{(X_i + X_j)}{2}, 1 \le i \le j \le N\right)$$

$$D = \frac{2\sqrt{\pi}}{N(N-1)}\sum_{i=1}^{N}\left(i - \frac{N+1}{2}\right)X_i$$

The sample variance of the finite population variance is defined as

$$\hat{t} = s_y^2 \tag{1.0}$$

its variance is given as:

$$Var(\hat{t}) = \gamma S_y^4\left(\beta_{2(y)} - 1\right) \tag{1.1}$$

[3] proposed a ratio-type estimator for the estimation of finite population variance when the population variance of auxiliary variable X is known. The bias and its mean squared error are given below:

$$\hat{S}_R^2 = s_y^2 \frac{S_x^2}{s_x^2} \tag{1.2}$$

$$Bias\left(\hat{S}_R^2\right) = \gamma S_y^4\left[\left(\beta_{2(x)} - 1\right) - \left(\lambda_{22} - 1\right)\right] \tag{1.3}$$

$$MSE\left(\hat{S}_R^2\right) = \gamma S_y^4\left[\left(\beta_{2(y)} - 1\right) + \left(\beta_{2(x)} - 1\right) - 2\left(\lambda_{22} - 1\right)\right] \tag{1.4}$$

[5] proposed a class of ratio type estimators for finite population variance by imposing Coefficient of variation and Coefficient of kurtosis on the work of Isaki (1983) as:

$$\hat{S}_{kc_1}^2 = s_y^2\left(\frac{S_x^2 + C_x}{s_x^2 + C_x}\right) \tag{1.5}$$

$$\hat{S}_{kc_2}^2 = s_y^2\left(\frac{S_x^2 + \beta_{x(2)}}{s_x^2 + \beta_{x(2)}}\right) \tag{1.6}$$

$$\hat{S}_{kc_3}^2 = s_y^2\left(\frac{S_x^2\beta_{x(2)} + C_x}{s_x^2\beta_{x(2)} + C_x}\right) \tag{1.7}$$

$$\hat{S}^2_{kc_4} = s_y^2\left(\frac{S_x^2 C_x + \beta_{x(2)}}{s_x^2 C_x + \beta_{x(2)}}\right) \quad (1.8)$$

$$Bias\left(\hat{S}^2_{kc_i}\right) = \gamma A_i S_y^2\left[A_i\left(\beta_{2(x)}-1\right)-\left(\lambda_{22}-1\right)\right], \text{ where i= 1,2,3,4} \quad (1.9)$$

$$MSE\left(\hat{S}^2_{kc_i}\right) = \gamma S_y^4\left[\left(\beta_{2(y)}-1\right)+A_i^2\left(\beta_{2(x)}-1\right)-2A_i\left(\lambda_{22}-1\right)\right], \text{ where i= 1,2,3,4}$$

(1.11)

where $A_1 = \frac{S_x^2}{s_x^2 + c_x}, A_2 = \frac{S_x^2}{s_x^2 + \beta_{2(x)}}, A_3 = \frac{S_x^2 \beta_{2(x)}}{s_x^2 \beta_{2(x)} + c_x}, A_4 = \frac{S_x^2 C_x}{s_x^2 \beta_{2(x)} + \beta_{2(x)}}$

[10] proposed a generalized modified ratio type estimator for finite population variance using the known parameters of the auxiliary variable as:

$$\hat{S}^2_{jG} = s_y^2\left(\frac{S_x^2 + \alpha w_i}{s_x^2 + \alpha w_i}\right)$$

(1.12)

$$Bias\left(\hat{S}^2_{jG}\right) = \gamma A_{jG} S_y^2\left[A_{jG}\left(\beta_{2(x)}-1\right)-\left(\lambda_{22}-1\right)\right]$$

(1.13)

$$MSE\left(\hat{S}^2_{jG}\right) = \gamma S_y^4\left[\left(\beta_{2(y)}-1\right)+A_{jG}^2\left(\beta_{2(x)}-1\right)-2A_{jG}\left(\lambda_{22}-1\right)\right]$$

(1.14)

where $A_{jG} = \frac{S_x^2}{s_x^2 + \alpha w_i}, \alpha = 1, w_1 = \frac{M_d}{C_x}$

[16] proposed a class of ratio type estimators for estimating finite population variance using known values of deciles of auxiliary variable as:

$$\hat{S}^2_{MS1} = s_y^2\left(\frac{S_x^2 + \left(D + D_1\right)}{s_x^2 + \left(D + D_1\right)}\right)$$

(1.15)

$$\hat{S}^2_{MS2} = s_y^2\left(\frac{S_x^2 + \left(D + D_2\right)}{s_x^2 + \left(D + D_2\right)}\right)$$

(1.16)

$$\hat{S}^2_{MS3} = s_y^2\left(\frac{S_x^2 + \left(D + D_3\right)}{s_x^2 + \left(D + D_3\right)}\right)$$

(1.17)

$$\hat{S}^2_{MS4} = s_y^2\left(\frac{S_x^2 + \left(D + D_4\right)}{s_x^2 + \left(D + D_4\right)}\right)$$

(1.18)

$$\hat{S}^2_{MS5} = s_y^2\left(\frac{S_x^2 + \left(D + D_5\right)}{s_x^2 + \left(D + D_5\right)}\right)$$

(1.19)

$$\hat{S}^2_{MS6} = s^2_y \left(\frac{S^2_x + (D + D_6)}{s^2_x + (D + D_6)} \right) \quad (1.21)$$

$$\hat{S}^2_{MS7} = s^2_y \left(\frac{S^2_x + (D + D_7)}{s^2_x + (D + D_7)} \right) \quad (1.22)$$

$$\hat{S}^2_{MS8} = s^2_y \left(\frac{S^2_x + (D + D_8)}{s^2_x + (D + D_8)} \right) \quad (1.23)$$

$$\hat{S}^2_{MS9} = s^2_y \left(\frac{S^2_x + (D + D_9)}{s^2_x + (D + D_9)} \right) \quad (1.24)$$

$$\hat{S}^2_{MS10} = s^2_y \left(\frac{S^2_x + (D + D_{10})}{s^2_x + (D + D_{10})} \right) \quad (1.25)$$

$$Bias\left(\hat{S}^2_{MSi}\right) = \gamma A_{MSi} S^2_y \left[A_{MSi} \left(\beta_{2(x)} - 1\right) - \left(\lambda_{22} - 1\right) \right], \qquad i = 1, 2, \ldots, 10 \quad (1.26)$$

$$MSE\left(\hat{S}^2_{MSi}\right) = \gamma S^4_y \left[\left(\beta_{2(y)} - 1\right) + A^2_{MSi} \left(\beta_{2(x)} - 1\right) - 2A_{MSi} \left(\lambda_{22} - 1\right) \right] \qquad i = 1, 2, \ldots, 10 \quad (1.27)$$

where $A_{MSi} = \frac{S^2_x}{S^2_x + a_i}$, $a_i = (D + D_i)$, $i = 1, 2, \ldots, 10$

$$A_{MS1} = \frac{S^2_x}{S^2_x + (D + D_1)}, A_{MS2} = \frac{S^2_x}{S^2_x + (D + D_2)}, A_{MS3} = \frac{S^2_x}{S^2_x + (D + D_3)}, A_{MS4} = \frac{S^2_x}{S^2_x + (D + D_4)},$$

$$A_{MS5} = \frac{S^2_x}{S^2_x + (D + D_5)}, A_{MS6} = \frac{S^2_x}{S^2_x + (D + D_6)}, A_{MS7} = \frac{S^2_x}{S^2_x + (D + D_7)}, A_{MS8} = \frac{S^2_x}{S^2_x + (D + D_8)},$$

$$A_{MS9} = \frac{S^2_x}{S^2_x + (D + D_9)}, A_{MS10} = \frac{S^2_x}{S^2_x + (D + D_{10})}$$

II. PROPOSED ESTIMATORS

Motivated by the work of [16], we proposed a class of new ratio-type estimators for estimating finite population variance using known information of first quartile, Downton's method,deciles and Mid-range, and Percentiles of auxiliary variable as:

$$\hat{S}^2_{J1} = s^2_y \left(\frac{S^2_x + (D_{10} + P_1)}{s^2_x + (D_{10} + P_1)} \right) \quad (2.1)$$

$$\hat{S}^2_{J2} = s^2_y \left(\frac{S^2_x + (D_{10} + P_5)}{s^2_x + (D_{10} + P_5)} \right) \quad (2.2)$$

$$\hat{S}^2_{MJ3} = s_y^2\left(\frac{S_x^2+\left(D_{10}+P_{10}\right)}{s_x^2+\left(D_{10}+P_{10}\right)}\right) \quad (2.3)$$

$$\hat{S}^2_{J4} = s_y^2\left(\frac{S_x^2+\left(D_{10}+P_{15}\right)}{s_x^2+\left(D_{10}+P_{15}\right)}\right) \quad (2.4)$$

$$\hat{S}^2_{J5} = s_y^2\left(\frac{S_x^2+\left(D_{10}+P_{20}\right)}{s_x^2+\left(D_{10}+P_{20}\right)}\right) \quad (2.5)$$

$$\hat{S}^2_{J6} = s_y^2\left(\frac{S_x^2+\left(D_{10}+P_{25}\right)}{s_x^2+\left(D_{10}+P_{25}\right)}\right) \quad (2.6)$$

$$\hat{S}^2_{J7} = s_y^2\left(\frac{S_x^2+\left(D_{10}+P_{30}\right)}{s_x^2+\left(D_{10}+P_{30}\right)}\right) \quad (2.7)$$

$$\hat{S}^2_{J8} = s_y^2\left(\frac{S_x^2+\left(D_{10}+P_{35}\right)}{s_x^2+\left(D_{10}+P_{35}\right)}\right) \quad (2.8)$$

$$\hat{S}^2_{J9} = s_y^2\left(\frac{S_x^2+\left(D_{9}+P_{40}\right)}{s_x^2+\left(D_{9}+P_{40}\right)}\right) \quad (2.9)$$

$$\hat{S}^2_{J10} = s_y^2\left(\frac{S_x^2+\left(MR+P_{45}\right)}{s_x^2+\left(MR+P_{45}\right)}\right)$$

(2.11)

$$\hat{S}^2_{J11} = s_y^2\left(\frac{S_x^2+\left(MR+P_{50}\right)}{s_x^2+\left(MR+P_{50}\right)}\right)$$

(2.12)

$$\hat{S}^2_{J12} = s_y^2\left(\frac{S_x^2+\left(MR+P_{55}\right)}{s_x^2+\left(MR+P_{55}\right)}\right)$$

(2.13)

$$\hat{S}^2_{J13} = s_y^2\left(\frac{S_x^2+\left(MR+P_{60}\right)}{s_x^2+\left(MR+P_{60}\right)}\right)$$

(2.14)

$$\hat{S}^2_{J14} = s_y^2\left(\frac{S_x^2+\left(MR+P_{65}\right)}{s_x^2+\left(MR+P_{65}\right)}\right)$$

(2.15)

$$\hat{S}^2_{J15} = s_y^2\left(\frac{S_x^2+\left(\bar{X}+P_{70}\right)}{s_x^2+\left(\bar{X}+P_{70}\right)}\right)$$

(2.16)

$$\hat{S}^2_{J16} = s_y^2\left(\frac{S_x^2+\left(\bar{X}+P_{75}\right)}{s_x^2+\left(\bar{X}+P_{75}\right)}\right)$$

(2.17)

$$\hat{S}^2_{J17} = s^2_y \left(\frac{S^2_x + (D + P_{80})}{s^2_x + (D + P_{80})} \right)$$
(2.18)

$$\hat{S}^2_{J18} = s^2_y \left(\frac{S^2_x + (D + P_{85})}{s^2_x + (D + P_{85})} \right)$$
(2.19)

$$\hat{S}^2_{J19} = s^2_y \left(\frac{S^2_x + (D + P_{90})}{s^2_x + (D + P_{90})} \right)$$
(2.21)

$$\hat{S}^2_{J20} = s^2_y \left(\frac{S^2_x + (D + P_{95})}{s^2_x + (D + P_{95})} \right)$$
(2.22)

$$\hat{S}^2_{J21} = s^2_y \left(\frac{S^2_x + (D + P_{99})}{s^2_x + (D + P_{99})} \right)$$
(2.23)

The proposed estimators can be written in a general form as:

$$\hat{S}^2_{Ji} = s^2_y \left(\frac{S^2_x + (D_k + P_j)}{s^2_x + (D_k + P_j)} \right), \; i = 1, 2, ..., 21 \quad j = 1, 5, 10, ..., 99.$$
(2.24)

2.1.1 Properties of the Proposed Estimators

Let $e_0 = \frac{s^2_y - S^2_y}{S^2_y}$ and $e_1 = \frac{s^2_x - S^2_x}{S^2_x}$ such that $s^2_y = S^2_y(1 + e_0)$ *and* $s^2_x = S^2_x(1 + e_1)$, where

$$\left. \begin{aligned} &E(e_0) = E(e_1) = 0, \; E(e_0^2) = \gamma(\beta_{2(y)} - 1) \\ &E(e_1^2) = \gamma(\beta_{2(x)} - 1), \; E(e_0 e_1) = \gamma(\lambda_{22} - 1) \end{aligned} \right\}$$
(2.25)

Expressing (2.24)in error terms, we have

$$\hat{S}^2_{Ji} = S^2_y (1 + e_0) \left(\frac{S^2_x + (D_k + P_j)}{S^2_x (1 + e_1) + (D_k + P_j)} \right)$$
(2.26)

$$\hat{S}^2_{Ji} = S^2_y (1 + e_0)(1 + A_{Ji} e_1)^{-1}$$
(2.27)

where $A_{Ji} = \frac{S^2_x}{S^2_x + (D_k + P_j)}$

Simplifying (2.27) up to first order approximation, it reduces to (2.28) as:

$$\hat{S}^2_{Ji} = S^2_y (1 + e_0)(1 - A_{Ji} e_1 + A^2_{Ji} e^2_1 ...)$$
(2.28)

Removing the brackets and subtracting both sides by S^2_y

$$\hat{S}_{Ji}^2 - S_y^2 = S_y^2\left(e_0 - A_{Ji}e_1 - A_{Ji}e_0e_1 + A_{Ji}^2e_1^2\right)$$

(2.29)

Taking Expectation of both sides of (2.29)

$$E\left(\hat{S}_{Ji}^2 - S_y^2\right) = S_y^2E\left(e_0 - A_{Ji}e_1 - A_{Ji}e_0e_1 + A_{Ji}^2e_1^2\right)$$

(2.31)

Applying the results of (2.25) obtaining the bias as

$$Bias\left(\hat{S}_{Ji}^2\right) = \gamma S_y^2\left[A_{Ji}^2\left(\beta_{2(x)}-1\right) - A_{Ji}\left(\lambda_{22}-1\right)\right]$$

(2.32)

To get the MSE, Squaring both sides of (2.29) and ignoring e of power two as:

$$E\left(\hat{S}_{Ji}^2 - S_y^2\right)^2 = S_y^4E\left(e_0 - A_{Ji}e_1\right)^2$$

(2.33)

Expanding, and taking expectation of (2.33)

$$MSE\left(\hat{S}_{Ji}^2\right) = S_y^4E\left(e_0^2 + A_{Ji}^2e_1^2 - 2A_{Ji}e_0e_1\right)$$

(2.34)

Applying the results of (2.25), obtaining $MSE\left(\hat{S}_{Ji}^2\right)$ as:

$$MSE\left(\hat{S}_{Ji}^2\right) = \gamma S_y^4\left[\left(\beta_{2(y)}-1\right) + A_{Ji}^2\left(\beta_{2(x)}-1\right) - 2A_{Ji}\left(\lambda_{22}-1\right)\right]$$

(2.35)

where

$$A_{J1} = \frac{S_x^2}{S_x^2+\left(D_{10}+P_1\right)},\ A_{J2} = \frac{S_x^2}{S_x^2+\left(D_{10}+P_5\right)},\ A_{J3} = \frac{S_x^2}{S_x^2+\left(D_{10}+P_{10}\right)},\ A_{J4} = \frac{S_x^2}{S_x^2+\left(D_{10}+P_{15}\right)},$$

$$A_{J5} = \frac{S_x^2}{S_x^2+\left(D_{10}+P_{20}\right)},\ A_{J6} = \frac{S_x^2}{S_x^2+\left(D_{10}+P_{25}\right)},\ A_{J7} = \frac{S_x^2}{S_x^2+\left(D_{10}+P_{30}\right)},\ A_{J8} = \frac{S_x^2}{S_x^2+\left(D_{10}+P_{35}\right)},$$

$$A_{J9} = \frac{S_x^2}{S_x^2+\left(D_9+P_{40}\right)},\ A_{J10} = \frac{S_x^2}{S_x^2+\left(MR+P_{45}\right)},\ A_{J11} = \frac{S_x^2}{S_x^2+\left(MR+P_{50}\right)},\ A_{J12} = \frac{S_x^2}{S_x^2+\left(MR+P_{55}\right)},$$

$$A_{J13} = \frac{S_x^2}{S_x^2+\left(MR+P_{60}\right)},\ A_{J14} = \frac{S_x^2}{S_x^2+\left(MR+P_{65}\right)},\ A_{J15} = \frac{S_x^2}{S_x^2+\left(\bar{X}+P_{70}\right)},\ A_{J16} = \frac{S_x^2}{S_x^2+\left(\bar{X}+P_{75}\right)},$$

$$A_{J17} = \frac{S_x^2}{S_x^2+\left(D+P_{80}\right)},\ A_{J18} = \frac{S_x^2}{S_x^2+\left(D+P_{85}\right)},\ A_{J19} = \frac{S_x^2}{S_x^2+\left(D+P_{90}\right)},\ A_{J20} = \frac{S_x^2}{S_x^2+\left(D+P_{95}\right)},$$

$$A_{J21} = \frac{S_x^2}{S_x^2+\left(Q_1+P_{99}\right)}$$

2.2 Efficiency Comparisons

Comparison of the proposed estimators with other existing estimators considered in the study with certain conditions to determine the estimators with higher precision.

The $\hat{S}_{Ji}^2$ - estimators of the finite population variance are more efficient than $Var(\hat{t})$ if,

$$MSE\left(\hat{S}_{Ji}^2\right) < Var\left(\hat{t}\right)$$

$$\left[\left(\beta_{2(y)}-1\right)+A_{Ji}^{2}\left(\beta_{2(x)}-1\right)-2A_{Ji}\left(\lambda_{22}-1\right)\right]<\left(\beta_{2(y)}-1\right)$$

(2.36)

The $\hat{S}_{Ji}^2$ - estimators of the finite population variance are more efficient than $\hat{S}_R^2$ if,

$$MSE\left(\hat{S}_{Ji}^2\right)<MSE\left(\hat{S}_R^2\right)$$

$$\left[A_{Ji}^{2}\left(\beta_{2(x)}-1\right)-2A_{Ji}\left(\lambda_{22}-1\right)\right]<\left[\left(\beta_{2(x)}-1\right)-2\left(\lambda_{22}-1\right)\right]$$

(2.37)

The $\hat{S}_{Ji}^2$ - estimators of the finite population variance are more efficient than $\hat{S}_{kc_i}^2$ if,

$$MSE\left(\hat{S}_{Ji}^2\right)<MSE\left(\hat{S}_{kc_i}^2\right)$$

$$\left[A_{Ji}^{2}\left(\beta_{2(x)}-1\right)-2A_{Ji}\left(\lambda_{22}-1\right)\right]<\left[A_{i}^{2}\left(\beta_{2(x)}-1\right)-2A_{i}\left(\lambda_{22}-1\right)\right]$$

(2.38)

The $\hat{S}_{Ji}^2$ - estimators of the finite population variance are more efficient than $\hat{S}_{jG}^2$ if,

$$MSE\left(\hat{S}_{Ji}^2\right)<MSE\left(\hat{S}_{jG}^2\right)$$

$$\left[A_{Ji}^{2}\left(\beta_{2(x)}-1\right)-2A_{Ji}\left(\lambda_{22}-1\right)\right]<\left[A_{jG}^{2}\left(\beta_{2(x)}-1\right)-2A_{jG}\left(\lambda_{22}-1\right)\right]$$

(2.39)

The $\hat{S}_{Ji}^2$ - estimators of the finite population variance are more efficient than $\hat{S}_{MSi}^2$ if,

$$MSE\left(\hat{S}_{MJi}^2\right)<MSE\left(\hat{S}_{MSi}^2\right)$$

$$\left[A_{Ji}^{2}\left(\beta_{2(x)}-1\right)-2A_{Ji}\left(\lambda_{22}-1\right)\right]<\left[A_{MSi}^{2}\left(\beta_{2(x)}-1\right)-2A_{MSi}\left(\lambda_{22}-1\right)\right]$$

(2.41)

When conditions (2.36), (2.37), (2.38), (2.39), and (2.41) are satisfied, we conclude that the proposed estimators $\left(\hat{S}_{Ji}^2\right)$ are more efficient than existing estimators.

2.2.1 Empirical Study

Empirical study is carried out to support the efficiency comparison stated above by considering a real life population as:

Data: [17]

Fixed capital (Auxiliary variable X)

Output of 80 factories (Study variable Y)

$N=80,\ n=20,\ S_x=8.4542,\ S_y=18.3569,\ C_x=0.7507,\ \bar{X}=11.2624,\ \bar{Y}=51.8264,\ \beta_{2(x)}=2.8664,$

$\beta_{2(y)}=2.2667,\ \beta_{1(x)}=1.05,\ \rho=0.9413,\ \lambda_{22}=2.2209,\ Q_1=9.318,\ C_y=0.3542,\ Q_2=7.5750,$

$Q_3=16.975,\ D=8.0138,\ D_1=3.6,\ D_2=4.6,\ D_3=5.9,\ D_4=6.7, D_5=7.5,\ D_6=8.5,\ D_7=14.8,\ D_8=18.1,$

$D_9=25.0,\ D_{10}=34.8, MR=17.955.\ P_1=2.44,\ P_5=4.35,\ P_{10}=5.9,\ P_{15}=6.63,\ P_{20}=7.45,\ P_{25}=7.8,$

$P_{30}=8.7,\ P_{35}=11.6,\ P_{40}=15.3,\ P_{45}=16.9,\ P_{50}=17.2,\ P_{55}=19.3,\ P_{60}=21.7,\ P_{65}=23.55,\ P_{70}=24.98,$

$P_{75}=25,\ P_{80}=26.95,\ P_{85}=27.8,\ P_{90}=29.7,\ P_{95}=30,\ P_{99}=34.85.$

Table 1: Bias, MSE and PRE of Existing and Proposed Estimators

Estimator	Bias	MSE	PRE
Sample variance	0	5393.89	100
Isaki (1983)	8.1569	2943.71	183.2344
Kadilar and Cingi (2006) 1	7.8297	2887.46	186.804
Subramani and Kumarapandiyan (2015)	4.6202	2389.24	225.7576
Bhat *et. al.* (2017) 1	4.1794	2330.10	231.4875
Bhat *et. al.* (2017) 2	3.9258	2297.74	234.7476
Bhat *et. al.* (2017) 3	3.6113	2258.56	238.8199
Bhat *et. al.* (2017) 4	3.4257	2236.84	241.1388
Bhat *et. al.* (2017) 5	3.2468	2215.55	243.456
Bhat *et. al.* (2017) 6	3.029	2191.71	246.1042
Bhat *et. al.* (2017) 7	1.8584	2078.86	259.4638
Bhat *et. al.* (2017) 8	1.3512	2041.82	264.1707
Bhat *et. al.* (2017) 9	0.4832	1999.23	269.7984
Bhat *et. al.* (2017) 10	0.4143	1999.24	269.797
Proposed Estimator (J1)	0.0529	1993.16	270.62
Proposed Estimator (J2)	-0.1210	1993.57	270.5644
Proposed Estimator (J3)	-0.2535	1995.33	270.3257
Proposed Estimator (J4)	-0.3787	1998.26	269.9293
Proposed Estimator (J5)	-0.2200	1994.76	270.403
Proposed Estimator (J6)	0.2879	1995.69	270.2769
Proposed Estimator (J7)	0.2572	1995.17	270.3474
Proposed Estimator (J8)	0.0515	1993.16	270.62
Proposed Estimator (J9)	-0.1649	1994.01	270.5047
Proposed Estimator (J10)	-0.3194	1996.71	270.1389
Proposed Estimator (J11)	0.1487	1993.79	270.5345
Proposed Estimator (J12)	0.1468	1993.77	270.5372
Proposed Estimator (J13)	0.2767	1995.50	270.3027
Proposed Estimator (J14)	0.1910	1994.24	270.4735
Proposed Estimator (J15)	0.0087	1993.07	270.6322
Proposed Estimator (J16)	-0.0190	1993.08	270.6309
Proposed Estimator (J17)	-0.1945	1994.39	270.4531
Proposed Estimator (J18)	0.0529	1993.16	270.62
Proposed Estimator (J19)	-0.1210	1993.57	270.5644
Proposed Estimator (J20)	-0.2535	1995.33	270.3257
Proposed Estimator (J21)	-0.3787	1998.26	269.9293

Table 1 shows the bias, MSE and PRE of the sample variance, [3], [5], [10], [16] and proposed estimators using a real life population. The resultsshown that all the proposed estimators having minimum MSE and higher PRE compared to the conventional estimators under simple random sampling scheme. This implies that the proposed estimators are more efficient than the estimators in the study.

III. RESULT AND DISCUSSION

A wide class of ratio estimators of finite population variance for estimation of study variable are developed. The results (Table 1) shown that the proposed estimators have minimum mean square error (MSE) and highest percentage relative efficiency (PRE) over the usual sample variance, [3], [5], [10], and [16]. The performance of the proposed estimators of finite population variance over sample variance, [3]ratio estimator and other selected existing estimators with real life population were examined. The Bias and Mean Square Error (MSE) of the proposed estimators were derived. The empirical study shown that the proposed class of estimators performed better than sample variance, [3], [5], [10], and [16].

IV. CONCLUSION

We proposed a wide class of ratio-type estimators $\hat{S}_{Ji}^2$ of the population variance S_y^2 of the study variable y when the information is available on the population parameters. The expressions for the bias and mean squared errors of the proposed class of ratio-type estimators has been derived up to first degree of approximation. We note that the MSE of the proposed estimators is smaller than the MSE of sample variance, [3], [5], [10], and [16] The results of the study shown that the proposed estimator are more efficient than the rest of the estimators considered in this research. At this junction, our recommendation is in the favour of the proposed ratio-type estimators for its use in practice for estimation of finite population variance.

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COMPETING INTERESTS

There is no competing interests associated with this study.

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